STATISTICS AND PROBABILITY

1. (a) Let C and G be the event "it's a cloudy day" and "Amos goes to school" respectively. $P(C)=\frac{3}{5}, P\left({}^{G}/_{C}\right)=\frac{7}{10}, P\left({}^{G}/_{C^1}\right)=\frac{1}{5}$

$$P\left({}^{C}/_{G^1}\right)=\frac{P(CnG^1)}{P(G^1)}=\frac{P(C)xP\left({}^{G^1}/_{C}\right)}{P(CnG^1)+P(C^1nG^1)}=\frac{\frac{3}{5}x\frac{3}{10}}{\frac{3}{5}x\frac{3}{10}+\frac{2}{5}x\frac{4}{5}}=\frac{9}{25}$$

(b) (i) For independent events $P(PnQ)=P(P)xP(Q)$

$$P(P)=P(PnQ)+P(PnQ^1)$$

$$(PnQ^1)=P(P)-P(P)xP(Q)=P(P)[1-P(Q)]=P(P)xP(Q^1)$$

Hence P and Q^1 are also independent events

(ii) $P(PuQ)=P(P)+P(Q)-P(PnQ)$

$$0.75=0.25+P(Q)-0.25xP(Q) \qquad P(Q)=\frac{0.5}{0.75}=\frac{2}{3}\ or\ 0.6667$$

2. $n=40, p=\frac{40}{100}=\frac{2}{5}, q=\frac{3}{5}$

Let x be a r.v "number of delegates supporting the salary increase

$x\sim B\left(450,\frac{2}{5}\right)$ Since n>20 then, $x\sim N(\mu,\sigma^2)$

But $\mu=450x\frac{2}{5}=180,\ \sigma^2=450x\frac{2}{5}x\frac{3}{5}=108$

(a) $P(x<150)=P\left(z<\frac{149.5-180}{\sqrt{108}}\right)$

$$=P(z<-2.935)=0.5-\emptyset(2.935)=0.5-0.0017=0.4983$$

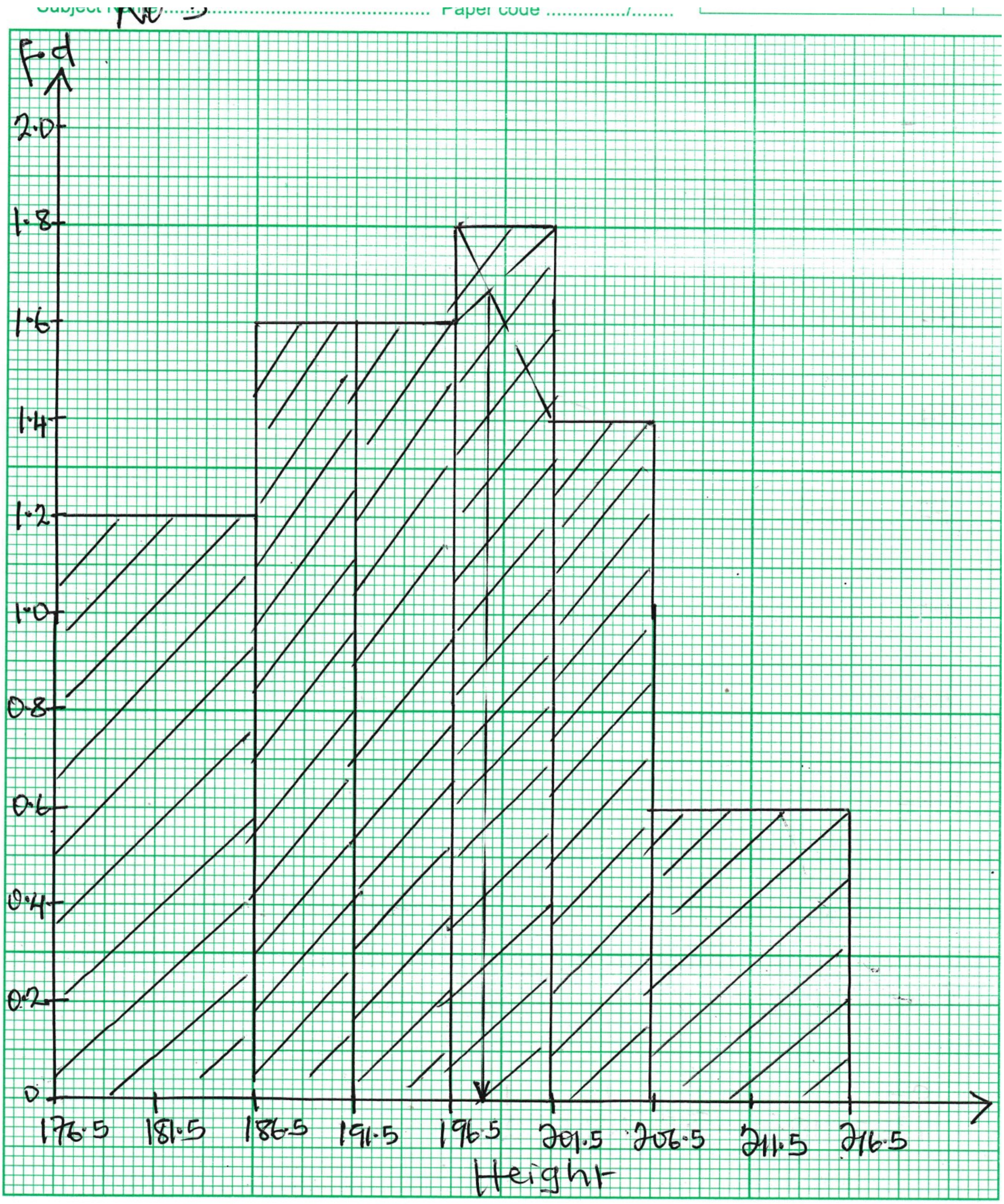
(b) $P(160<x<170)=P\left(\frac{159.5-180}{\sqrt{108}}<z<\frac{170.5-180}{\sqrt{108}}\right)$

$$=P(-1.973<z<-0.914)=\emptyset(0.914)-\emptyset(1.973)=0.1804-0.0243$$
$$=0.1561$$

3.

Class boundaries	Mid-point, x	f	fx	fx^2	Freq. density
176.5 - 186.5	181.5	12	2178	395307	1.2
186.5 – 191.5	189	8	1512	285768	1.6
191.5 – 196.5	194	8	1552	301088	1.6
196.5 – 201.5	199	9	1791	356409	1.8
201.5 – 206.5	204	7	1428	291312	1.4
206.5 – 216.5	211.5	6	1269	268393.5	0.6
		$\sum f=50$	$\sum fx=9730$	$\sum fx^2=1898277.5$	

(a) Modal class is (197 – 201)



(a) (i) Mean, $\bar{x} = \frac{9730}{50} = 194.6cm$

(ii) Standard deviation $= \sqrt{\frac{1898277.5}{50} - (194.6)^2} = 9.8178cm$

(iii) Mode$=196.5 + \frac{0.2}{0.2+0.4}x5 = 198.1667cm$

4. (i) $P(^B/_A) = \frac{P(BnA)}{P(A)} = 0.2$

But $PAuB) = P(A) + P(B) - P(AnB)$

$0.6 = x + y - P(AnB) \quad \therefore P(AnB) = (x + y) - 0.6$

Hence $\frac{(x+y)-0.6}{x} = 0.2 \quad \therefore 4x + 5y = 3 \ldots\ldots\ldots (i)$

(ii) $P(BuC) = P(B) + P(C) \quad \rightarrow 0.9 = y + (x + y) \quad \therefore 10x + 20y = 9 \ldots\ldots\ldots (ii)$

(iii) Solving (i) and (ii) simultaneously gives $x = \frac{1}{2} and\ y = \frac{1}{5}$

5. Let A and G be events the person has been involved in an accident and wears glasses respectively. $P(A) = \frac{9}{30}, P(^G/_A) = \frac{5}{9}$ and $P\left(^G/_{A^1}\right) = \frac{1}{3}$

(a) $P(G) = P(AnG) + P(A^1nG) \quad \rightarrow P(G) = \frac{9}{30}x\frac{5}{9} + \frac{21}{30}x\frac{1}{3} = \frac{8}{15}$

(b) $P(^A/_G) = \frac{P(AnG)}{P(G)} = \frac{\frac{9}{30}x\frac{5}{9}}{\frac{8}{15}} = \frac{5}{16}$

6. (a) (i) $P(^A/_B) = \frac{P(AnB)}{P(B)} \quad 0.2 = \frac{P(AnB)}{0.1} \quad \therefore P(AnB) = 0.02$

Hence $P(AuB) = P(A) + P(B) - P(AnB) = 0.7 + 0.1 - 0.02 = 0.78$

(ii) $P\left(^A/_{B^1}\right) = \frac{P(AnB^1)}{P(B^1)} = \frac{P(A)-P(AnB)}{P(B^1)} = \frac{0.7-0.02}{0.9} = 0.7556$

(b) Let A, B and G be events for picking “box A”, “box B” and a “green ball” respectively. $P(A) = \frac{1}{6}, P(B) = \frac{5}{6}, P(^G/_A) = \frac{3}{5} and\ P(^G/_B) = \frac{1}{5}$

$$P(^A/_G) = \frac{P(AnG)}{P(G)} = \frac{P(AnG)}{P(AnG)+P(BnG)} = \frac{\frac{1}{6}x\frac{3}{5}}{\frac{1}{6}x\frac{3}{5}+\frac{5}{6}x\frac{1}{5}} = \frac{3}{8}$$

7.

x	1	2	3	4
F(x)	0.14	0.47	0.79	1.00
P(X=x)	0.14	0.33	0.32	0.21

(i) $P(2 < x \leq 4) = F(4) - F(2) = 1 - 0.47 = 0.53$

or $P(2 < x \leq 4) = P(X = 3) + P(X = 4) = 0.32 + 0.21 = 0.53$

(ii) Median, $F(x) \geq 0.5$ Hence median = 3

(iii) $P\left(^{x\,<\,3}/_{2\,\leq\,x\,<\,4}\right) = \frac{P(x<3\ n\ 2\leq x<4)}{P(2\leq x<4)} = \frac{P(x=2)}{P(x=2)+P(x=3)} = \frac{0.33}{0.33+0.32} = 0.5077$

(iv) $Mean = 1x0.14 + 2x0.33 + 3x0.32 + 4x0.21 = 2.6$

8. (i) $At\ x = a, \quad \frac{2}{13}(a+1) = \frac{2}{13}(5-a) \quad \therefore a = 2$

$\int_0^2 \frac{2}{13}(x+1)dx + \int_2^b \frac{2}{13}(5-x)dx = 1$

$\frac{2}{13}\left[\left(\frac{x^2}{2}+x\right)_0^2 + \left(5x-\frac{x^2}{2}\right)_2^b\right] = 1$

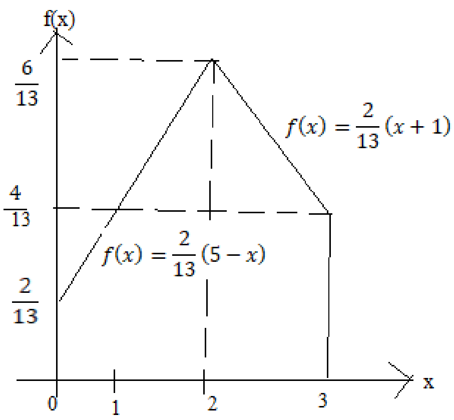
$\frac{2}{13}\left[(2+2) + \left(5b-\frac{b^2}{2}\right) - (10-2)\right] = 1$

$b^2 - 10b + 21 = 0 \ \therefore b = 7\ or\ b = 3$

Check: $b = 7, \int_2^7 \frac{2}{13}(5-x)dx = \frac{2}{13}\left[5x-\frac{x^2}{2}\right]_2^7 = \frac{2}{13}\left[\left(35-\frac{49}{2}\right)-(10-2)\right] = \frac{5}{13}$

$b = 3, \int_2^3 \frac{2}{13}(5-x)dx = \frac{2}{13}\left[5x-\frac{x^2}{2}\right]_2^3 = \frac{2}{13}\left[\left(15-\frac{9}{2}\right)-(10-2)\right] = \frac{5}{13}$

Hence $a = 2, b = 3\ or\ 7$



$0 \le x \le 2$

$f(0) = \frac{2}{13}(0+1) = \frac{2}{13}, \qquad f(2) = \frac{2}{13}(2+1) = \frac{6}{13}$

$2 \le x \le 3$

$f(2) = \frac{2}{13}(5-2) = \frac{6}{13}, \qquad f(3) = \frac{2}{13}(5-3) = \frac{4}{13}$

(ii) $E(x) = \int_0^2 \frac{2}{13}x(x+1)dx + \int_2^3 \frac{2}{13}x\,(5-x)dx$

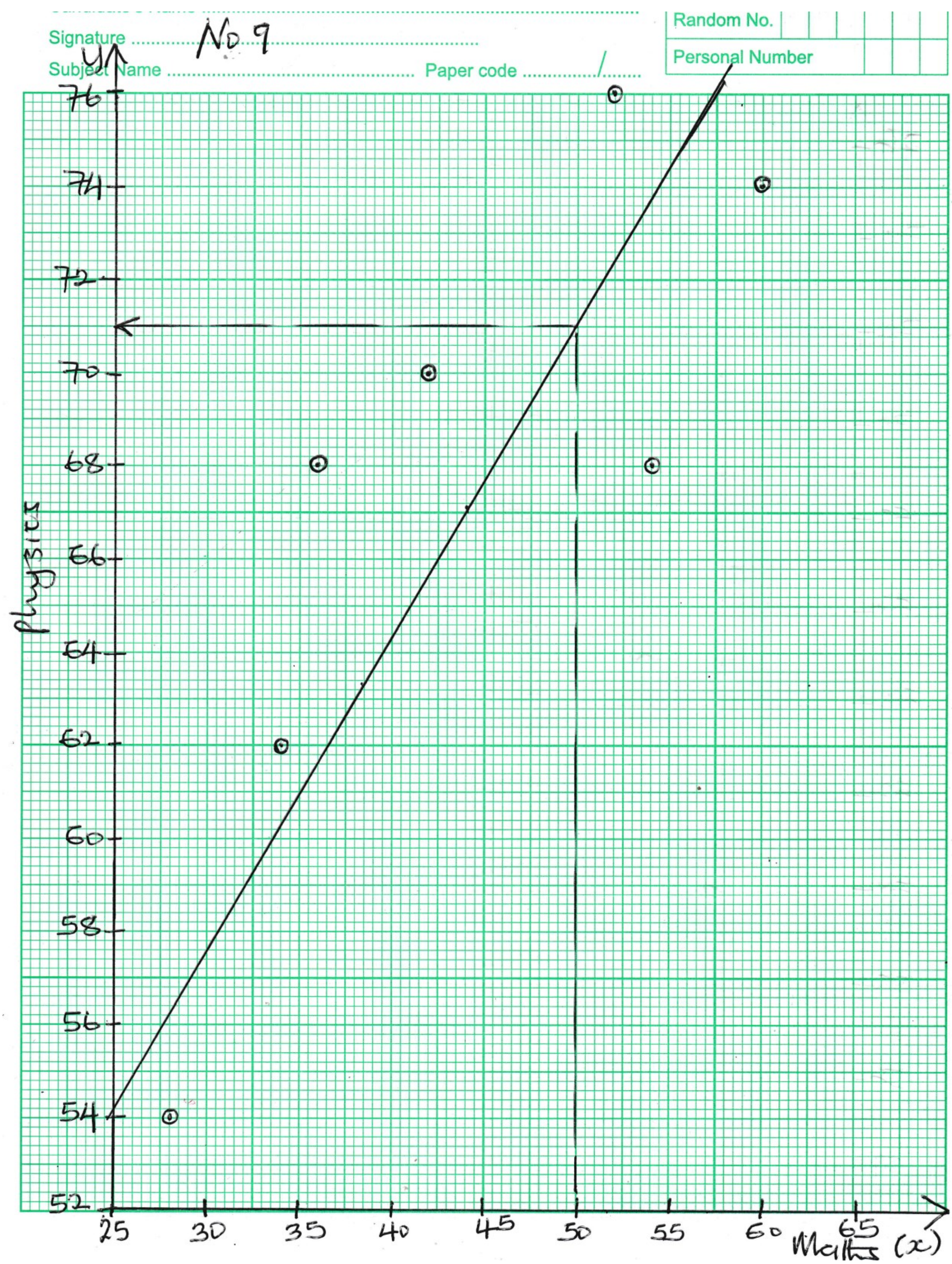
$= \frac{2}{13}\left[\left(\frac{x^3}{3}+\frac{x^2}{2}\right)_0^2 + \left(\frac{5x^2}{2}-\frac{x^3}{3}\right)_2^3\right]$

$= \frac{2}{13}\left[\left(\frac{2^3}{3}+\frac{2^2}{2}\right) + \left(\frac{5(3)^2}{2}-\frac{3^3}{3}\right) - \left(\frac{5(2)^2}{2}-\frac{(2)^3}{3}\right)\right] = \frac{5}{3} = 1\frac{2}{3}$

(iii) $P(x < 2.5) = \int_0^2 \frac{2}{13}(x+1)dx + \int_2^{2.5} \frac{2}{13}(5-x)dx$

$= \frac{2}{13}\left[\left(\frac{x^2}{2}+x\right)_0^2 + \left(5x-\frac{x^2}{2}\right)_2^{2.5}\right]$

$= \frac{2}{13}\left[(2+2) + \left(5x2.5-\frac{2.5^2}{2}\right) - (10-2)\right] = 0.8269$



9. (a) (i) There is a positive relation between Physics and Maths

(ii) When Maths = 50, Physics = 71

Maths(x)	Physics(y)	R_x	R_y	D^2
28	54	7	7	0
34	62	6	6	0
36	68	5	4.5	0.25
42	70	4	3	1
52	76	3	1	4
54	68	2	4.5	6.25
60	74	1	2	1
				$\sum D^2 = 12.5$

$$\rho = 1 - \frac{6x12.5}{7(7^2 - 1)} = 0.7768$$

Since |ρ|=0.7768>0.75(table value), then there is significance of Maths on Physics performance at 5% level of significance based on 7 pairs of observations

10. (a) Sample mean $\bar{x} = \frac{2.57}{10} = 0.257$ Population mean $\hat{\mu} = \bar{x} = 0.257$

Sample variance $s^2 = \frac{0.6610}{10} - (0.257)^2 = 0.000051$

Population Variance $\hat{\sigma}^2 = \frac{10}{9} x 0.000051 = 0.00057$

$95\%\ Confidence\ limits = 0.257 \pm 1.96x \frac{0.000057}{\sqrt{10}} = (0.256996,\ 0.257003)$

(b) Let x be a r.v “ weight of ball bearings $x \sim N(25, 4^2)$

$$P(24.12 < \bar{x} < 26.73) = P\left(\frac{24.12 - 25}{\frac{4}{\sqrt{16}}} < z < \frac{26.73 - 25}{\frac{4}{\sqrt{16}}}\right)$$

$= P(-0.88 < z < 1.73)$

$= \emptyset(1.73) + \emptyset(0.88)$

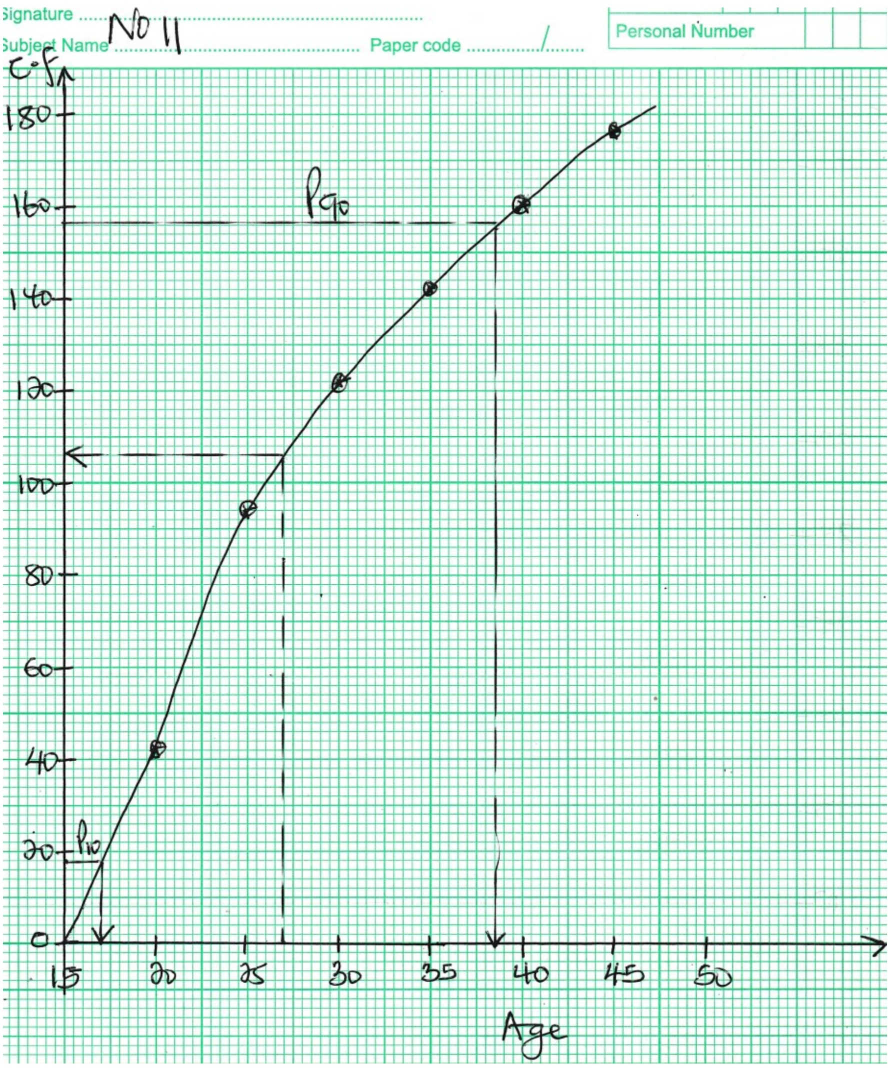
$= 0.4582 + 0.3106$

$= 0.7688$

11.

Class boundaries	Mid-point, x	f	fx	c.f
15 – 20	17.5	42	735	42
20 - 25	22.5	52	1170	94
25 - 30	27.5	28	770	122
30 – 35	32.5	20	650	142
35 – 40	37.5	18	675	160
40 - 45	42.5	16	680	176
		$\sum f = 176$	$\sum fx = 4680$	

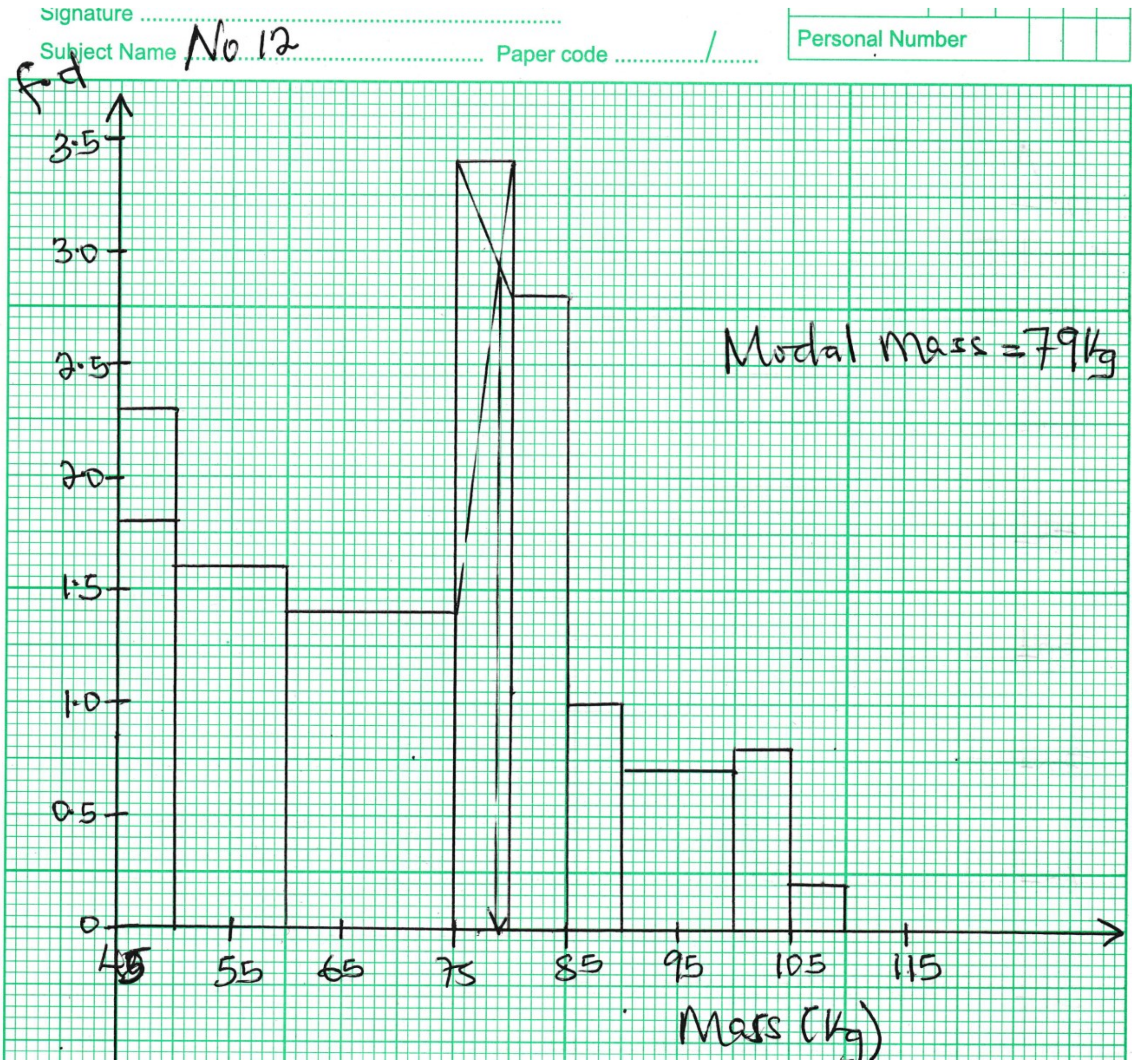
(a) Mean $=\frac{4680}{176} = 26.5909$ Median $=25 + \frac{\left(\frac{176}{2}-94\right)}{28}x5 = 23.9286$



(b) (i) Proportion more than 27 years = $\frac{106}{176} = \frac{53}{88}$

(ii) 80% Central limits = $[38.5, 17.0\ ]$

12.



Class boundaries	Mid-point, x	f	fx	fx^2	Freq. density
45 - 50	47.5	9	427.5	20306.25	1.8
50 - 60	55	16	880	48400	1.6
60 - 75	67.5	21	1417.5	95681.25	1.4
75 - 80	77.5	17	1317.5	102106.25	3.4
80 - 85	82.5	14	1155	95287.5	2.8
85 - 90	87.5	5	437.5	38281.25	1.0
90 - <100	95	7	665	63175	0.7
100 - <105	102.5	4	410	42025	0.8
105 - <110	107.5	1	107.5	11556.25	0.2
		$\sum f = 94$	$\sum fx = 6817.5$	$\sum fx^2 = 516818.75$	

$$Standard\ deviation\ = \frac{516818.75}{94} - \left(\frac{6817.5}{94}\right)^2 = 15.4261$$

13. Mean, E(x) = $\int_a^b \frac{1}{b-a} x dx$

$= \left[\frac{x^2}{2(b-a)}\right]_a^b = \frac{b^2 - a^2}{2(b-a)} = \frac{(b-a)(b+a)}{2(b-a)} = \frac{(a+b)}{2}$

E(x^2) $= \int_a^b \frac{1}{(b-a)} x^2$ dx

$= \left[\frac{x^3}{3(b-a)}\right]_a^b = \frac{b^3-a^3}{3(b-a)} = \frac{(b-a)(b^2+ab+a^2)}{3(b-a)} = \frac{b^2+ab+b^2}{3}$

Hence Variance = E(x^2) – $(E(x))^2$

$= \frac{b^2+ab+a^2}{3} - \left(\frac{a+b}{2}\right)^2$

$= \frac{b^2+ab+a^2}{3} - \frac{b^2+2ab+a^2}{4}$

$= \frac{4b^2+4ab+4a^2-3b^2-6ab-3a^2}{12} = \frac{b^2-2ab+a^2}{12} = \frac{(b-a)^2}{12}$

b) Mean: $1 = \frac{b+a}{2}$

$b + a = 2$………………. (i)

Variance: $\frac{3}{4} = \frac{(b-a)^2}{12}$

$(b-a)^2 = 9$

$b - a \ = 3$……………………(ii)

Solving (i) and (ii) simultaneously gives;

b = 2.5 and **a = -0.5**

(i) P(X<0) $= \int_{-0.5}^{0} \frac{1}{(2.5--0.5)} dx \ = \frac{1}{3}[X]_{-0.5}^{0} = \frac{(0--0.5)}{3} = \mathbf{\frac{1}{6}}$

(ii) $P(x > a + \sigma) = \frac{1}{4}$ But $\sigma = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{4}$

$$\int_{a+\frac{\sqrt{3}}{4}}^{2.5} \frac{1}{3} dx = \frac{1}{4} \ \rightarrow \left[\frac{x}{3}\right]_{a+\frac{\sqrt{3}}{4}}^{2.5} = \frac{1}{4} \ \rightarrow \frac{2.5-(a+\frac{\sqrt{3}}{4})}{3} = \frac{1}{4} \ \rightarrow a + \frac{\sqrt{3}}{4} = 2.5 - \frac{3}{4}$$

$$\therefore a = 0.8840$$

14. (i) **Price relatives**= $\left(\frac{P_{2011}}{P_{2010}} \boldsymbol{x100}\right)$

Aspirin: $\frac{125}{80}x100 = \mathbf{156.25}$, Panadol: $\frac{90}{100}x100 = \mathbf{90}$,

Quinine: $\frac{75}{55}x100 = \mathbf{136.3636}$, Coartem: $\frac{100}{90}x100 = \mathbf{111.1111}$

(ii) Simple aggregate price index $= \frac{\sum P_{2011}}{\sum P_{2010}} x100 = \frac{(125+90+75+100)}{(80+100+55+90)} x100$

$$= \frac{390}{325} x100$$

$$= 120$$

(iii) Weighted price Index $= \frac{\sum\left(\frac{P_{2011}}{P_{2010}}\right)w}{\sum W} x100 = \frac{\left(\frac{125}{80}x45 + \frac{90}{100}x90 + \frac{75}{55}x10 + \frac{100}{90}x10\right)}{45+90+10+10} x100$

$$= 113.5871$$

Hence the prices in 2011 increased by 13.5871%

(iv) Weighted aggregate Price Index $= \frac{\sum(P_{2011})w}{\sum(P_{2010})w} x100$

$$= \frac{(125x45+90x90+75x10+100x10)}{(80x40+100x70+55x8+90x10)} x100$$

$$= \frac{15475}{11540} x100 = 134.0988$$

Hence the price in 2011 increased by 34.0988%

MECHANICS

15. (a) $F = ma \qquad 0.2a = 8t\tilde{\boldsymbol{i}} - 4t^2\tilde{\boldsymbol{j}} + 2(3 - t^2)\tilde{\boldsymbol{k}}$

$$a = (40t\tilde{\boldsymbol{i}} - 20t^2\tilde{\boldsymbol{j}} + 10(3 - t^2\boldsymbol{k})$$

Hence $v = \int a dt \quad \rightarrow v = \int(40t\tilde{\boldsymbol{i}} - 20t^2\tilde{\boldsymbol{j}} + 10(3 - t^2\boldsymbol{k})dt$

$$v = 20t^2\boldsymbol{i} - \frac{20}{3}t^3\boldsymbol{j} + 10(3t - \frac{t^3}{3})\boldsymbol{k} + c$$

When t=0s, c= 0. $\therefore\ v = 20t^2\boldsymbol{i} - \frac{20}{3}t^3\boldsymbol{j} + 10(3t - \frac{t^3}{3})\boldsymbol{k}$

(b) $r(t) = r(o) + v.t$

$$r(t) = (-10\,i + 12j - 4k) + \left[20t^2\boldsymbol{i} - \frac{20}{3}t^3\boldsymbol{j} + 10\left(3t - \frac{t^3}{3}\right)\boldsymbol{k}\right]\boldsymbol{t}$$

$$r(t) = (20t^3 - 10)i + \left(12 - \frac{20}{3}t^4\right)j + \left(30t^2 - \frac{10}{3}t^4 - 4\right)k$$

When $t = 2s, \qquad r(2) = (20x8 - 10)i + \left(12 - \frac{20}{3}x16\right)j + \left(30x4 - \frac{10}{3}x2^4 - 4\right)k$

$$r(2) = 150i - \frac{284}{3}j + \frac{188}{3}k$$

$\therefore Distance, |r(2)| = \sqrt{(150^2 + (-\frac{284}{3})^2 + \left(\frac{188}{3}\right)^2}$ = 188.1194m

16. (i) $F = \begin{pmatrix} \frac{t}{2} - 1 \\ t - 3 \end{pmatrix} + \begin{pmatrix} \frac{t}{2} + 2 \\ \frac{t}{2} - 4 \end{pmatrix} + \begin{pmatrix} t - 3 \\ 3t - 2 \end{pmatrix} = \begin{pmatrix} 2t - 2 \\ \frac{9t}{2} - 9 \end{pmatrix} N$

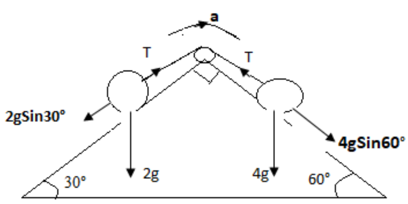
Work done = F.$\overline{AB}$ = $\begin{pmatrix} 2x1 - 2 \\ \frac{9x1}{2} - 9 \end{pmatrix} . \left[\begin{pmatrix} 7 \\ -9 \end{pmatrix} - \begin{pmatrix} 4 \\ -5 \end{pmatrix}\right] = \begin{pmatrix} 0 \\ -\frac{9}{2} \end{pmatrix} . \begin{pmatrix} 3 \\ -14 \end{pmatrix} = 0 + 63 = 63J$

(ii) F=ma, $4a = \begin{pmatrix} 2x2 - 2 \\ \frac{9x2}{2} - 9 \end{pmatrix} \quad 4a = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad a = \begin{pmatrix} 0.5 \\ 0 \end{pmatrix}$

Hence $|a| = \sqrt{(0.5)^2 + 0}\ \therefore |a| = 0.5m/s^2$

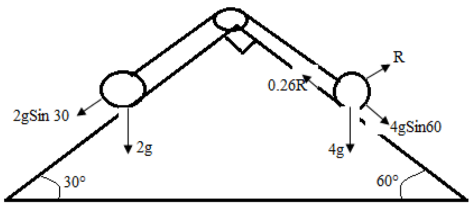
(iii) Power=F.V $\ Power = \begin{pmatrix} 2x2 - 2 \\ \frac{9x2}{2} - 9 \end{pmatrix} . \begin{pmatrix} 3 \\ -6 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} . \begin{pmatrix} 3 \\ -6 \end{pmatrix} = 6 + 0 = 6W$

17. (a)



4kg mass: 4gsin60°-T = 4a ……….(1)
2kg mass: T – 2gSin30°=2a ………(2)
Adding (1) and (2)
4gsin60° - 2gSin30° =6a
a= 4.0247 m/s²
Tension: T = 2(4.0247 +9.8Sin30°)
= **17.8494N**

(b)



4kg mass: 4gsin60°- 0.25R-T = 4a, But R = 4gCos60°

4gsin60° - 0.25x4gCos60°-T = 4a…….(3)

2kg mass: T – 2gSin30°=2a ………(4)

Adding (3) and (4)

4gsin60° - 0.25x4gCos60°- 2gSin30° =6a

a= 3.2080 m/s²

18. (a) Resultant force, $F = \begin{pmatrix}1\\-4\end{pmatrix} + \begin{pmatrix}3\\6\end{pmatrix} + \begin{pmatrix}-9\\1\end{pmatrix} + \begin{pmatrix}5\\-3\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$

Moment, M $= \begin{vmatrix}3 & -1\\1 & -4\end{vmatrix} + \begin{vmatrix}2 & 2\\3 & 6\end{vmatrix} + \begin{vmatrix}-1 & -1\\-9 & 1\end{vmatrix} + \begin{vmatrix}-3 & 4\\5 & -3\end{vmatrix}$

$= (-12 + 1) + (12 - 6) + (-1 - 9) + (9 - 20)$

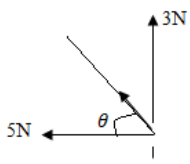
= -26Nm.

Since the resultant, F = 0 and the moment, M=26Nm in Clockwise direction, then the forces reduce to a couple

(b) The fourth force, $F_4 = \begin{pmatrix}5\\-3\end{pmatrix} N$, hence the resultant force of the remaining forces is

$F = \begin{pmatrix}-5\\3\end{pmatrix}$

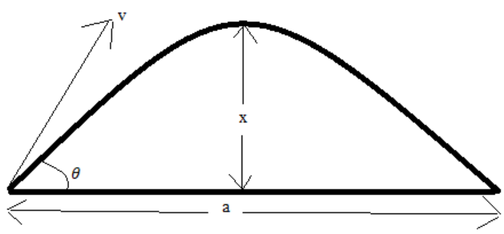
Magnitude, $|F| = \sqrt{(-5)^2 + 3^2} = \sqrt{34}$ = 5.8310N



$\tan\theta = \frac{3}{5} \quad \therefore \theta = 30.96°$

Hence the resultant force is 5.8310N acting at angle of 300.96° or N29.04°W

19. (a)



Range, $a = (v\cos\theta).\frac{2v\sin\theta}{g} = \frac{2v^2\sin\theta\cos\theta}{g} = \frac{v^2\sin 2\theta}{g}$

$\therefore \sin 2\theta = \frac{ag}{v^2} \qquad \rightarrow \cos\theta = \frac{\sqrt{v^4-(ag)^2}}{v^2}$

Maximum height, $x = \frac{v^2\sin^2\theta}{2g}$ But $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$

$$x = \frac{v^2.\frac{1}{2}(1 - \frac{\sqrt{v^4-(ag)^2}}{v^2})}{2g}$$

$$4gx = v^2 - \sqrt{v^4 - (ag)^2})$$

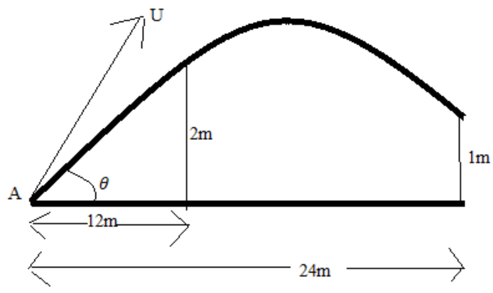
$$(4gx - v^2)^2 = (\sqrt{v^4 - (ag)^2})^2$$

$$16g^2x^2 - 8gv^2x + v^4 = v^4 - a^2g^2$$

$$16g^2x^2 - 8gv^2x + a^2g^2 = 0$$

$$16gx^2 - 8v^2x + a^2g = 0$$

(b)



From equation of a trajectory; $y = x\tan\theta - \frac{gx^2(1+\tan^2\theta)}{2u^2}$

$$\textbf{\textit{Case 1}}: \quad 2 = 12\tan\theta - \frac{10x12^2(1+\tan^2\theta)}{2U^2}$$

$$1 = 6\tan\theta - \frac{360(1+\tan^2\theta)}{U^2}$$

$$(1+\tan^2\theta) = \frac{(6\tan\theta - 1)U^2}{360} \ldots\ldots\ldots\ldots\ldots..(i)$$

$$\textbf{\textit{Case 2}}: 1 = 24\tan\theta - \frac{10x24^2(1+\tan^2\theta)}{2U^2}$$

$$1 = 24\tan\theta - \frac{2880(1+\tan^2\theta)}{U^2} \quad \rightarrow (1+\tan^2\theta) = \frac{(24\tan\theta-1)U^2}{2880} \ldots\ldots\ldots\ldots\ldots\ldots..(ii)$$

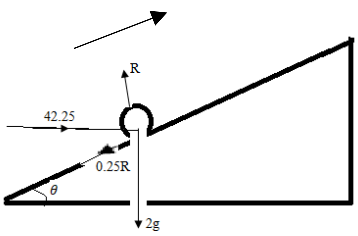
Equating (i) and (ii)

$$\frac{(6\tan\theta - 1)U^2}{360} = \frac{(24\tan\theta - 1)U^2}{2880}$$

$$8(6\tan\theta - 1) = (24\tan\theta - 1)$$

$$48\tan\theta - 24\tan\theta - 7 = 0 \quad \rightarrow \tan\theta = \frac{7}{24} \quad \therefore \theta = 16.3°$$

20. (a)



Using F =ma,

Along the inclined plane:

$42.25Cos\theta - 0.25R - 2gSin\theta = 2a$, But R = 2gCos$\theta$

$42.25Cos\theta - 0.25x2gCos\theta - 2gSin\theta = 2a$, Sin$\theta = \frac{3}{5}$ hence Cos$\theta = \frac{4}{5}$

$$a = \frac{42.25x\frac{4}{5} - 2x10(0.25x\frac{4}{5} + \frac{3}{5})}{2} = 8.9m/s^2$$

PQ = 15m = s, u =0m/s

From $v^2 = u^2 + 2as$, Hence $v^2 = 0 + 2x8.9x15 \quad \therefore v = 16.3401m/s$

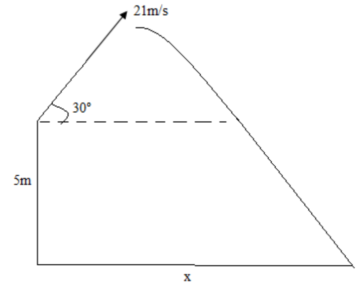
(b) Volume per second, $v = speedxarea = 7.5x\frac{10}{10{,}000} = 7.5x10^{-3}m^3/s$

Mass per second, $m = densityxvolume\ per\ second = 1000x7.5x10^{-3} = 7.5kg/s$

Work done per second = $K.E + P.E = \frac{1}{2}x7.5x7.5^2 + 7.5x9.8x10 = 945.9375J/s$

Hence Power done = 945.9375W

21. (a)



Vertical displacement: $-5 = (21sin30°)t - \frac{1}{2}x9.8xt^2$

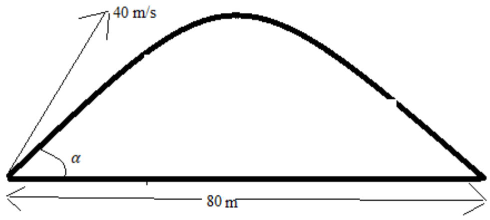
$$-5 = \left(\frac{21}{2}\right)t - \frac{9.8}{2}t^2 \quad \rightarrow 9.8t^2 - 21t - 10 = 0$$

$$t = 2.544s \; or \; t = -0.401s$$

Hence $t = 2.544s$

Horizontal displacement: $x = (21cos30)x2.544 = 46.2665m$

(b)



Time of flight: $T = \frac{2x40Sin\alpha}{10} \quad \rightarrow sin\alpha = \frac{T}{8} \ldots\ldots(i)$

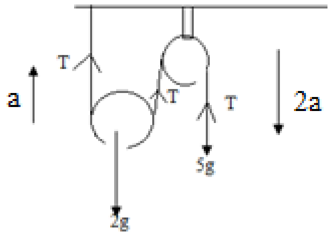
$$Range: 80 = (40cos\alpha)T \quad \rightarrow cos\alpha = \frac{2}{T} \ldots\ldots\ldots(ii)$$

$$(Sin\alpha)^2 + (cos\alpha)^2 = 1$$

$$\left(\frac{T}{8}\right)^2 + \left(\frac{2}{T}\right)^2 = 1$$

$$\frac{T^4 + 256}{64T^2} = 1 \quad \therefore T^4 - 64T^2 + 256 = 0$$

22.



(i) 2kg Pulley: $2T - 2g = 2a$............(i)

5Kg mass: 5g – T = 5x2a ➡ T = 5(g – 2a)..............(ii)

Putting eqn(ii) in eqn(i)

2x5(g -2a) – 2g = 22a ➡ 22a=8g

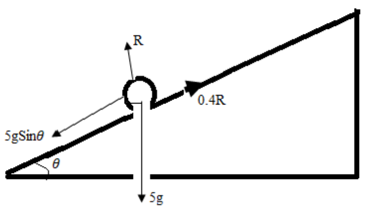
$\therefore a = \frac{8x9.8}{22} = \mathbf{3.5636m/s^2}$ $(acceleration\ of\ 2kg\ pulley)$ and

Acceleration of 5Kg mass = 2x3.5636 = **7.1272m/s^2**

(ii) From eqn(ii); T =5(9.8 – 2x3.5636) = **13.364N**

(iii) $s = 0x1.5 + \frac{1}{2}x3.5636x1.5^2 = 4.0091m$

23. (a)



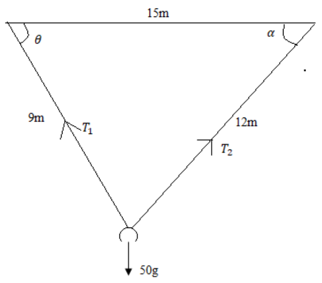
$5gSin\theta - 0.4R = 5a$, But $R = 5gCos\theta$

$$a = \frac{5x9.8(\frac{4}{5} - 0.4x\frac{3}{5})}{5} = 5.488m/s^2$$

From $v^2 = u^2 + 2as$

$$v^2 = 0 + 2x5.488x8 \quad \therefore v = 9.3906m/s$$

(b)



Resolving vertically: $T_1Sin\theta + T_2Sin\alpha = 50g$……………………(i)

Resolving Horizontally: $T_1Cos\theta = T_2Cos\alpha$ …………………………..(ii)

Using Cosine rule: $12^2 = 9^2 + 15^2 - 2x9x15Cos\theta$

$$Cos\theta = \frac{9^2+15^2-12^2}{2x9x15} = \frac{3}{5} \text{ Hence } Sin\theta = \frac{4}{5} \quad \therefore \theta = 53.1°$$

Also $\quad 9^2 = 12^2 + 15^2 - 2x12x15Cos\alpha$

$$Cos\alpha = \frac{12^2+15^2-9^2}{2x12x15} = \frac{4}{5} \text{ Hence } Sin\alpha = \frac{3}{5} \quad \therefore \alpha = 36.9°$$

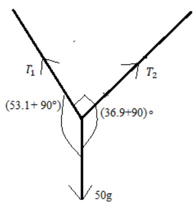
From (i) $T_1x\frac{4}{5} + T_2x\frac{3}{5} = 50g \longrightarrow \quad 4T_1 + 3T_2 = 250g \ldots\ldots\ldots\ldots..(iii)$

(ii) $T_1x\frac{3}{5} = T_2x\frac{4}{5} \longrightarrow \quad T_1 = \frac{4}{3}T_2$ ……….(iv)

Using (iv) in (iii) $4x\frac{4}{3}T_2 + 3T_2 = 250g \qquad T_2 = \frac{3x250x9.8}{25} = 294\boldsymbol{N}$

Hence $T_1 = \frac{4}{3}x294 = \boldsymbol{392N}$

Method 2: Using Lami's theorem

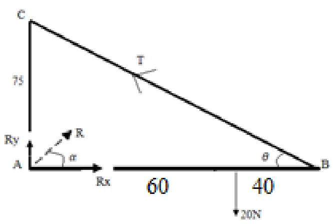


$$\frac{T_1}{Sin(36.1 + 90°)} = \frac{T_2}{Sin(53.1 + 90°)} = \frac{50g}{Sin90°}$$

$$\therefore T_1 = \frac{50x9.8xsin126.9°}{sin90°} = 391.8455 \approx 392N$$

Also; $T_2 = \frac{50x9.8xsin143.1°}{sin90°} = 294.2059 \approx 294N$

24.



Taking moments about point A;

$$Tx100Sin\theta = 20x60 \quad \text{But } Sin\theta = \frac{75}{125} = \frac{3}{5} \; and \; Cos\theta = \frac{4}{5}$$

$$Tx100x\frac{3}{5} = 20x60 \qquad \therefore T = \frac{5x1200}{300} = 20N$$

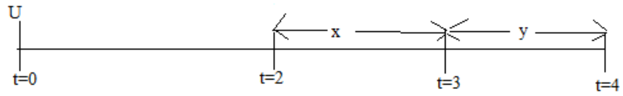
Resolving:

Vertically: $TSin\theta + R_y = 20 \;\rightarrow 20x\frac{3}{5} + R_y = 20 \;\rightarrow R_y = 8N$

Horizontally: $R_x = TCos\theta \quad \rightarrow R_x = 20x\frac{4}{5} = 16N$

$$Magnitude, |R| = \sqrt{8^2 + 16^2} = 17.8885N \qquad Direction, \quad Tan\alpha = \frac{16}{8} \quad \alpha = 63.4°$$

Hence the magnitude of the reaction at A is 17.8885N acting at angle of 63.4° to the horizontal.



25. (a)

Let U be the initial velocity.

$Using\ S = ut + \frac{1}{2}at^2$

$$\textbf{In 3rd second}: x = \left(ux3 + \frac{1}{2}a(3)^2\right) - \left(ux2 + \frac{1}{2}a(2)^2\right) = u + \frac{5}{2}a$$

$$a = \frac{2}{5}(x - u) \ldots\ldots\ldots.(i)$$

$$\textbf{In 4th second}: y = \left(ux4 + \frac{1}{2}a(4)^2\right) - \left(ux3 + \frac{1}{2}a(3)^2\right) = u + \frac{7}{2}a$$

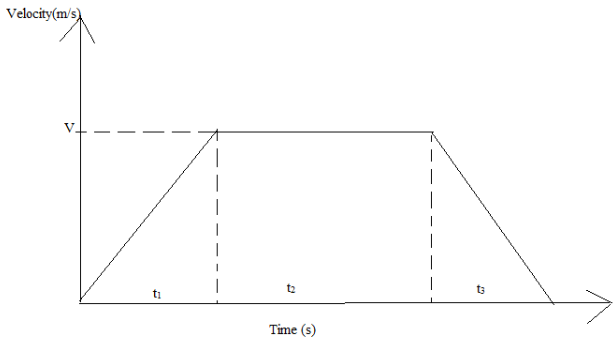
$$a = \frac{2}{7}(y - u) \ldots\ldots\ldots.(ii)$$

$Equating (i) and\ (ii)$

$\frac{2}{5}(x - u) = \frac{2}{7}(y - u) \quad \rightarrow x - u = \frac{5}{7}(y - u) \quad \rightarrow 7x - 7u = 5y - 5u$

$\rightarrow 2u = 7x - 5y \quad \therefore \boldsymbol{u = \frac{1}{2}(7x - 5y)}$

(b).



$$Acceleration\ region: x = \frac{1}{2}.V.t_1 \quad \rightarrow t_1 = \frac{2x}{V}$$

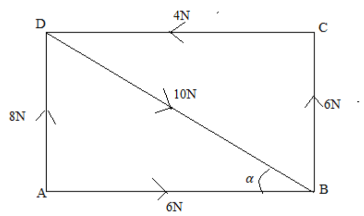
$$Constant\ velocity\ region: 3x = V.t_2 \rightarrow t_2 = \frac{3x}{V}$$

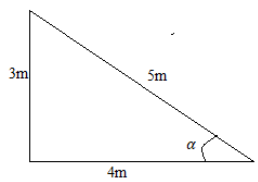
$$Deceleration\ regio: 2x = \frac{1}{2}.V.t_3 \quad \rightarrow t_3 = \frac{4x}{V}$$

$$Total\ time: T = t_1 + t_2 + t_3$$

$$T = \frac{2x}{V} + \frac{3x}{V} + \frac{4x}{V} = \boldsymbol{\frac{9x}{V}}s$$

26.

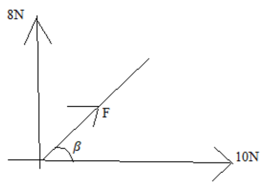




$$Sin\alpha = \frac{3}{5}, Cos\alpha = \frac{4}{5}$$

(i) Resultant force, $F = \begin{pmatrix}6\\0\end{pmatrix} + \begin{pmatrix}0\\6\end{pmatrix} + \begin{pmatrix}-4\\0\end{pmatrix} + \begin{pmatrix}0\\8\end{pmatrix} + \begin{pmatrix}10Cos\alpha\\-10Sin\alpha\end{pmatrix} = \begin{pmatrix}2 + 10x\frac{4}{5}\\14 - 10x\frac{3}{5}\end{pmatrix} = \begin{pmatrix}10\\8\end{pmatrix}$

Magnitude $|F| = \sqrt{10^2 + 8^2} = \boldsymbol{12.8062N}$



Direction: $tan\beta = \frac{8}{10}$ $\therefore \beta = \mathbf{38.7°}$

Hence the magnitude of resultant force is 12.8062N acting at angle of 38.7° to the horizontal.

(ii) Let the resultant force act at a point (x,y)

$$Resultant\ moment = \begin{vmatrix} x & y \\ 10 & 8 \end{vmatrix} = 8x - 10y \dots\dots\dots(i)$$

$$Taking\ moments\ about\ point\ B, M = 4x3 - 8x4 = -20Nm \dots\dots(ii)$$

$$Equating\ the\ moment\ of\ resultant\ force\ and\ sum\ of\ moment\ of\ individaul\ forces$$

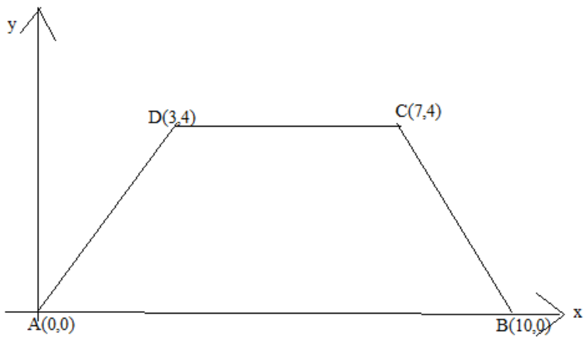
$8x - 10y = -20$

$Hence\ \mathbf{4x - 5y + 10}$
$= \mathbf{0}\ is\ the\ equation\ of\ the\ line\ of\ action\ of\ the\ resultant\ force$

(iii) It cuts AB produced when y = 0, $4x - 5(0) + 10 = 0$ $\rightarrow 4x = -10$ $\rightarrow x = -2.5m$

Hence the line of action cuts AB at a distance of 2.5m from B on the left.

27. (a) $weight\ \propto lenght\ \rightarrow w\alpha\ l\ \rightarrow w = gl\ where\ g\ is\ constant$



$AB = \sqrt{(10-0)^2 + (0-0)^2} = 10\ units$

$BC = \sqrt{(7-10)^2 + (4-0)^2} = 5\ units$

$CD = \sqrt{(7-3)^2 + (4-4)^2} = 4\ units$

$AD = \sqrt{(3-0)^2 + (4-0)^2} = 5\ units$

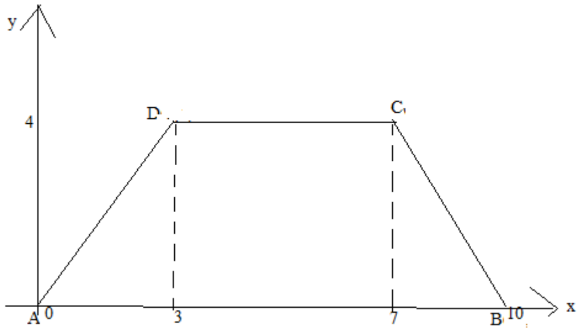
Note: Each length is uniform, so the resultant mass is at the centre

$$(10+5+4+5)g\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 10g\begin{pmatrix}5\\0\end{pmatrix} + 5g\begin{pmatrix}8.5\\2\end{pmatrix} + 4g\begin{pmatrix}5\\4\end{pmatrix} + 5g\begin{pmatrix}1.5\\2\end{pmatrix}$$

$$24\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}120\\36\end{pmatrix} \quad \therefore \begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}5\\1.5\end{pmatrix}$$

Hence centre of gravity is at a point (**5, 1.5**)

(b) (i) If ABCD is a lamina, $weight \propto area \rightarrow w \alpha A \rightarrow w = gA$

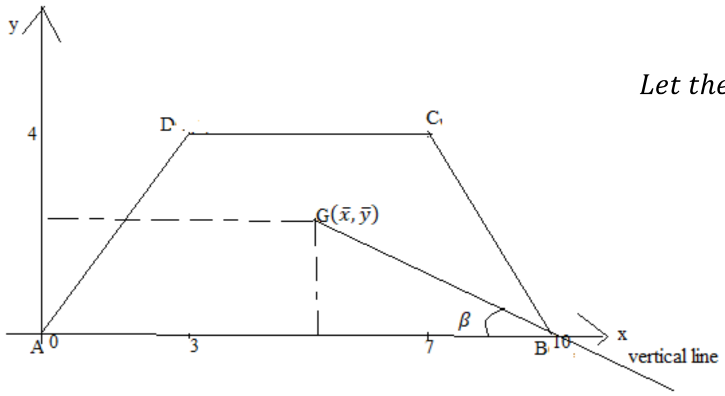


Body	Area (sq. units)	Weight	Position of c.o.g from origin
1st triangle	$\frac{1}{2}x3x4 = 6$	6g	$\left(\frac{2}{3}x3, \frac{1}{3}x4\right) = (2, \frac{4}{3})$
Rectangle	4x4 = 16	16g	$\left(3 + \frac{1}{2}x4, \frac{1}{2}x4\right) = (5,2)$
2nd triangle	$\frac{1}{2}x3x4 = 6$	6g	$\left(7 + \frac{1}{3}x3, \frac{1}{2}x4\right) = (8,2)$
Whole body	28	28g	$(\bar{x}, \bar{y})$

$$28g\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 6g\begin{pmatrix}2\\ 4/3\end{pmatrix} + 16g\begin{pmatrix}5\\ 2\end{pmatrix} + 6g\begin{pmatrix}8\\ 2\end{pmatrix}$$

$$28\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}140\\ 52\end{pmatrix} \quad \therefore \begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}5\\ 1.8571\end{pmatrix}$$

Hence centre of gravity is at a point (**5, 1.8571**)



Let the angle made by AB with vertical be β

$$Tan\beta = \frac{(10-5)}{1.8571} \quad \therefore \beta = 69.6°$$

28. (a) $F = ma \quad (2\boldsymbol{i} + 3\boldsymbol{j} - 4\boldsymbol{k}) = 5\boldsymbol{a} \quad \therefore \boldsymbol{a} = \frac{1}{5}(2\boldsymbol{i} + 3\boldsymbol{j} - 4\boldsymbol{k})\boldsymbol{m/s^2}$

(b) $v = \int a dt \quad \rightarrow v = \int \frac{1}{5}(2\boldsymbol{i} + 3\boldsymbol{j} - 4\boldsymbol{k})\boldsymbol{dt} \quad v = \frac{1}{5}(2t\boldsymbol{i} + 3t\boldsymbol{j} - 4t\boldsymbol{k}) + \boldsymbol{c}$

$When\ t = 0, v = 0 \ \rightarrow c = 0$

$Hence$ when t=3s, $v = \frac{1}{5}(2x3\boldsymbol{i} + 3x3\boldsymbol{j} - 4x3\boldsymbol{k}) \ \rightarrow \boldsymbol{v} = \frac{\mathbf{1}}{\mathbf{5}}(\mathbf{6}\boldsymbol{i} + \mathbf{9}\boldsymbol{j} - \mathbf{12}\boldsymbol{k})\boldsymbol{m/s}$

(c) $r(t) = r(0) + v.t \ \rightarrow r(3) = \begin{pmatrix}-2\\1\\0\end{pmatrix} + \begin{pmatrix}6/5\\9/5\\-12/5\end{pmatrix} x3 = \begin{pmatrix}8/5\\32/5\\-36/5\end{pmatrix}$ m

Distance from the origin $|r(3)| = \sqrt{\left(\frac{8}{5}\right)^2 + \left(\frac{32}{5}\right)^2 + \left(\frac{-32}{5}\right)^2} = 9.1913m$

29. $\hat{V}c = \begin{pmatrix}20sin40°\\20cos40°\end{pmatrix} \ \hat{V}w = \begin{pmatrix}10cos60°\\-10sin60°\end{pmatrix}$

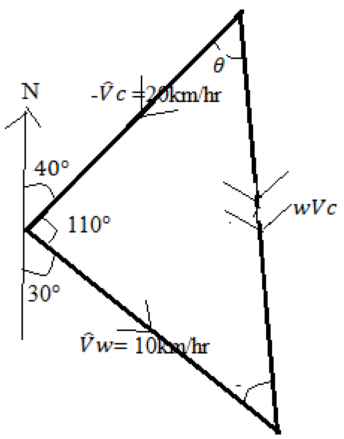
$wVc = Vw - Vc = \begin{pmatrix}10cos60°\\-10sin60°\end{pmatrix} - \begin{pmatrix}20sin40°\\20cos40°\end{pmatrix} = \begin{pmatrix}-7.8558\\-23.9811\end{pmatrix}$

$$|wVc| = \sqrt{(-7.8558)^2 + (-23.9811)^2} = 25.2350km/hr$$

Direction $tan\theta = \frac{23.9811}{7.8558} \quad \therefore \theta = 71.9°$

Hence $|wVc| = 25.2350kmhr^{-1}\ acting\ at\ angle\ of\ 71.9°\ to\ the\ horizontal\ or$ $S18.1°\ W\ or 198.1°$

Method 2:



$$|wVc|^2 = 20^2 + 10^2 - 2x10x20Cos110°$$

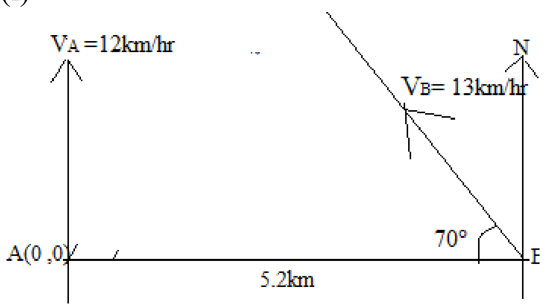
$$|wVc| = 25.2350\ km/hr$$

$$\frac{Sin\theta}{10} = \frac{Sin110°}{25.2350}$$

$$\theta = sin^{-1}\left(\frac{10Sin110°}{25.2350}\right) = 21.9°$$

$Direction\ is\ S18.1°\ W\ or\ 198.1°$

b(i)



$$V_A = \begin{pmatrix}0\\12\end{pmatrix}, \quad V_B = \begin{pmatrix}-13Cos70°\\13sin70°\end{pmatrix}$$

$$r_A = \begin{pmatrix}0\\0\end{pmatrix} + \begin{pmatrix}0\\12\end{pmatrix}t = \begin{pmatrix}0\\12t\end{pmatrix}$$

$$r_B = \begin{pmatrix}5.2\\0\end{pmatrix} + \begin{pmatrix}-13Cos70°\\13sin70°\end{pmatrix}t = \begin{pmatrix}5.2 - 13tCos70°\\13tsin70°\end{pmatrix}$$

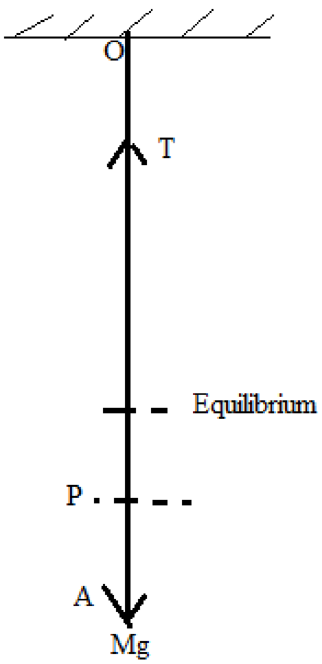
$_{B}r_{A} = r_B - r_A = \begin{pmatrix} 5.2 - 13tCos70° \\ 13tsin70° \end{pmatrix} - \begin{pmatrix} o \\ 12t \end{pmatrix} = \begin{pmatrix} 5.2 - (13Cos70°)t \\ (13sin70° - 12)t \end{pmatrix}$

(ii) At 8:30, $t = 30min - \frac{1}{2}hrs$

$_{B}r_{A}\left(t = \frac{1}{2}\right) = \begin{pmatrix} 5.2 - (13Cos70°)x0.5 \\ (13sin70° - 12)x0.5 \end{pmatrix} = \begin{pmatrix} 2.9769 \\ 0.1080 \end{pmatrix}$

$Distance\ between\ A\ and\ B = \sqrt{(2.9769)^2 + (0.1080)^2} = 2.9789km$

30. (i)



*Let **A** be the lowest point , **P** the general point and **y** the extension of the string.*

Using the principle of conservation of energy

$$(K.E + P.E + E.P.E)_o = (K.E + P.E + E.P.E)_A$$

$$(0 + mg(L + y) + 0) = 0 + 0 + \frac{1}{2}\frac{4Mg}{L}y^2$$

$$(L + y) = \frac{2}{L}y^2$$

$$2y^2 - Ly - L^2 = 0$$

$$(2y9 + L)(y - L) = 0$$

$$y = -\frac{L}{2}\ or\ y = L$$

Hence greatest distance below O $= \boldsymbol{L + L = 2L}$

Let the extension at equilibrium be $\boldsymbol{e}$

At equilibrium: $\boldsymbol{Mg = T = \left(\frac{4Mg}{L}\right)e \quad \rightarrow e = \frac{L}{4}}$

When displaced through a distance, $\boldsymbol{x}$, restoring force,

$$\boldsymbol{Force = Mg - T^1, where\ T^1\ is\ new\ tension}$$

$$\boldsymbol{Ma = Mg - \frac{4Mg}{L}(x + e)}$$

$$\boldsymbol{Ma = Mg - \frac{4Mg}{L}x - \frac{4Mg}{L}e; \quad But: Mg = \left(\frac{4Mg}{l}\right)e}$$

$$\boldsymbol{Ma = Mg - \left(\frac{4Mg}{L}\right)e - Mg = -\left(\frac{4Mg}{L}\right)e}$$

$$\therefore a = -\left(\frac{4g}{L}\right)x$$

Compare with $a = -\omega^2 x \quad \rightarrow \omega^2 = \frac{4g}{L} \quad \rightarrow \omega = \sqrt{\frac{4g}{L}} = 2\sqrt{\frac{g}{L}}$

$$Amplitude, A = L - \frac{L}{4} = \frac{3L}{4}$$

$$Acceleration\ at\ L = A\omega^2 = \frac{3L}{4} x \frac{4g}{L} = 3g$$

(ii) $When\ acceleration\ is\ zero, velocity\ is\ maximum,$

$$V_{max} = Aw = \frac{3L}{4} x 2\sqrt{\frac{g}{L}} = \frac{3}{2}\sqrt{gL}$$

Hence speed is $1.5\sqrt{gL}$

NUMERICAL METHODS

31. (a) (i)

48.11	x	46.93
7.621	8.614	9.043

$$\frac{9.043-7.621}{46.93-48.11}=\frac{8.614-7.621}{x-48.11}$$

$$\therefore x = 47.2861$$

(ii)

51.07	50.24	48.11
y	4.116	7.621

$$\frac{7.621-y}{48.11-51.07}=\frac{7.621-4.116}{48.11-50.24} \quad \therefore y = 2.7503$$

$$Hence\ f^{-1}(51.07) = 2.7503$$

(b)

$h = \frac{2-1}{5} = 0.2$ Let $y = \frac{x^2}{x^2+1}$

x	y_0, y_6	$y_1,\ldots y_5$
1.0	0.5	
1.2		0.5902
1.4		0.6622
1.6		0.7191
1.8		0.7642
2.0	0.8	
Sum	**1.3**	**2.7357**

$$\int_1^2 \frac{x^2}{x^2+1}dx \approx \frac{1}{2}x0.2(1.3+2x2.7357) = 0.677\ \ (3d.ps)$$

(c) $\int_1^2 \frac{x^2}{x^2+1}dx = \int_1^2 \left(1-\frac{1}{x^2+1}\right)dx = [x - tan^{-1}(x)]_1^2 =$

$$= \left(2 - tan^{-1}(2)\right) - \left(1 - tan^{-1}(1)\right) == 0.678$$

Error can be reduced by increasing the number of strips or sub-intervals

32. $h = \frac{2-1}{6} = \frac{1}{6}$ Let $y = (x-1)lnx$

x	y_0, y_6	$y_1,\ldots y_5$
1.0	0.00000	
$7/6$		0.02569
$4/3$		0.09589
$3/2$		0.20273
$5/3$		0.34055
$11/6$		0.50511
2.0	0.69315	
Sum	**0.69315**	**1.16997**

$$\int_1^2 (x-1)lnxdx \approx \frac{1}{2}x\frac{1}{6}(0.69315 + 2x1.16997) \qquad = 0.2528\ \ (4d.ps)$$

Exact value: $\int_1^2 (x-1)lnxdx = \frac{1}{2}[lnx(x^2-2x)]_1^2 - \int_1^2 \left(\frac{x^2}{2}-x\right).\frac{1}{x}dx$

$$= \frac{1}{2}[lnx(x^2-2x)]_1^2 - \int_1^2 \left(\frac{x}{2}-1\right)dx$$

$$= \frac{1}{2}\left[\left(lnx(x^2-2x)\right)\right]_1^2 - \left[\frac{x^2}{4}-x\right]_1^2 = (0-0) - \left(-1 - -\frac{3}{4}\right) = 0.25$$

$$Maximum\ error\ = |Exact\ value - approx.value| = |0.25 - 0.2528| = 0.0028$$

33. Error, $e = (x+\Delta x)^2(y+\Delta y) - x^2y$

$$= (x^2 + 2x\Delta x + (\Delta x)^2)(y+\Delta y) - x^2y$$
$$= x^2y + 2xy\Delta x + y(\Delta x)^2 + x^2\Delta y + 2x\Delta y\Delta x + \Delta y(\Delta x)^2 - x^2y$$
$$= 2xy\Delta x + y(\Delta x)^2 + x^2\Delta y + 2x\Delta y\Delta x + \Delta y(\Delta x)^2$$

Assumption: If $\Delta x \ll x$ and $\Delta y \ll y, then$ $(\Delta x)^2 \approx \Delta y\Delta x \approx \Delta y(\Delta x)^2 \approx 0$,then;

$$e = 2xy\Delta x + x^2\Delta y$$

Absolute error, $|e| = |2xy\Delta x + x^2\Delta y|$

$$|e| \le |2xy\Delta x| + |x^2\Delta y|$$

Hence maximum error, $|\boldsymbol{e}| = |\boldsymbol{2xy\Delta x}| + |\boldsymbol{x^2\Delta y}|$

$|e| = |2(2.8)(1.44)0.0016| + |2.8^2(0.008)| = 0.0756$

Upper limit = $2.8^2 x 1.44 + 0.0756$ = 11.3652
Lower limit = $2.8^2 x 1.44 - 0.0756$ = 11.2140

34. (a) Let $f(x) = 3x^3 - x + 5$

$$f(-1.4) = 3(-1.4)^3 - (-1.4) + 5 = -1.832$$
$$f(-1.2) = 3(-1.2)^3 - (-1.2) + 5 = 1.016$$

Since $f(-1.4) < 0$ and $f(-1.2) > 0, then -1.4 < x_{root} < -1.2$

(b)

-1.4	x_o	-1.2
-1.832	0	1.016

$$\frac{1.016 - -1.832}{-1.2 - -1.4} = \frac{1.016 - 0}{x_o - -1.4} \qquad \therefore x_o = -1.329$$

(c) $f(x) = 3x^3 - x + 5 \;\; \rightarrow f^1(x) = 9x^2 - 1$

$$x_{n+1} = x_n - \frac{f(x)}{f^1(x)} = x_n - \frac{(3x^3 - x + 5)}{9x^2 - 1}$$

$$x_1 = -1.329 - \frac{(3(-1.329)^3 - (-1.329) + 5)}{9(-1.329)^2 - 1} = -1.281$$

$$x_2 = -1.281 - \frac{(3(-1.281)^3 - (-1.281) + 5)}{9(-1.281)^2 - 1} = -1.279$$

$$|e| = |-1.279 - -1.281| = 0.002 < 0.005$$

Hence $\boldsymbol{x_{root} \approx -1.28}$

35. (a) (i) $\Delta l = 0.5,\ \Delta w = 0.005$

(ii) $Maximum\ area = 8.5x4.265 = 36.25$

$Minimum\ area = 7.5x4.255 = 31.91$

$Range = [31.91,\quad 36.25]$

(b) $volume\ of\ a\ cylinder, v = \frac{1}{3}r^2h$

Error in volume, $\Delta v = \frac{1}{3}(r + \Delta r)^2(h + \Delta h) - \frac{1}{3}r^2h$

$$= \frac{1}{3}(r^2 + 2r\Delta r + (\Delta r)^2).(h + \Delta h) - \frac{1}{3}r^2h$$

$$= \frac{1}{3}[(r^2h + 2rh\Delta r + (\Delta r)^2h) + r^2\Delta h + 2r\Delta r\Delta h + (\Delta r)^2\Delta h - r^2h]$$

$$= \frac{1}{3}[(2rh\Delta r + (\Delta r)^2h) + r^2\Delta h + 2r\Delta r\Delta h + (\Delta r)^2\Delta h]$$

$Assumptions: If\ \Delta r \ll r\ and\ \Delta h \ll h, then (\Delta r)^2 \approx \Delta r\Delta h \approx (\Delta r)^2\Delta h \approx 0$

$$\Delta v = \frac{1}{3}[2rh\Delta r + r^2\Delta h]$$

$$Relative\ error: \frac{\Delta v}{v} = \frac{\frac{1}{3}[2rh\Delta r + r^2\Delta h]}{\frac{1}{3}r^2h} = 2\frac{\Delta r}{r} + \frac{\Delta h}{h}$$

Absolute relative error $\left|\frac{\Delta v}{v}\right| = \left|2\frac{\Delta r}{r} + \frac{\Delta h}{h}\right|$

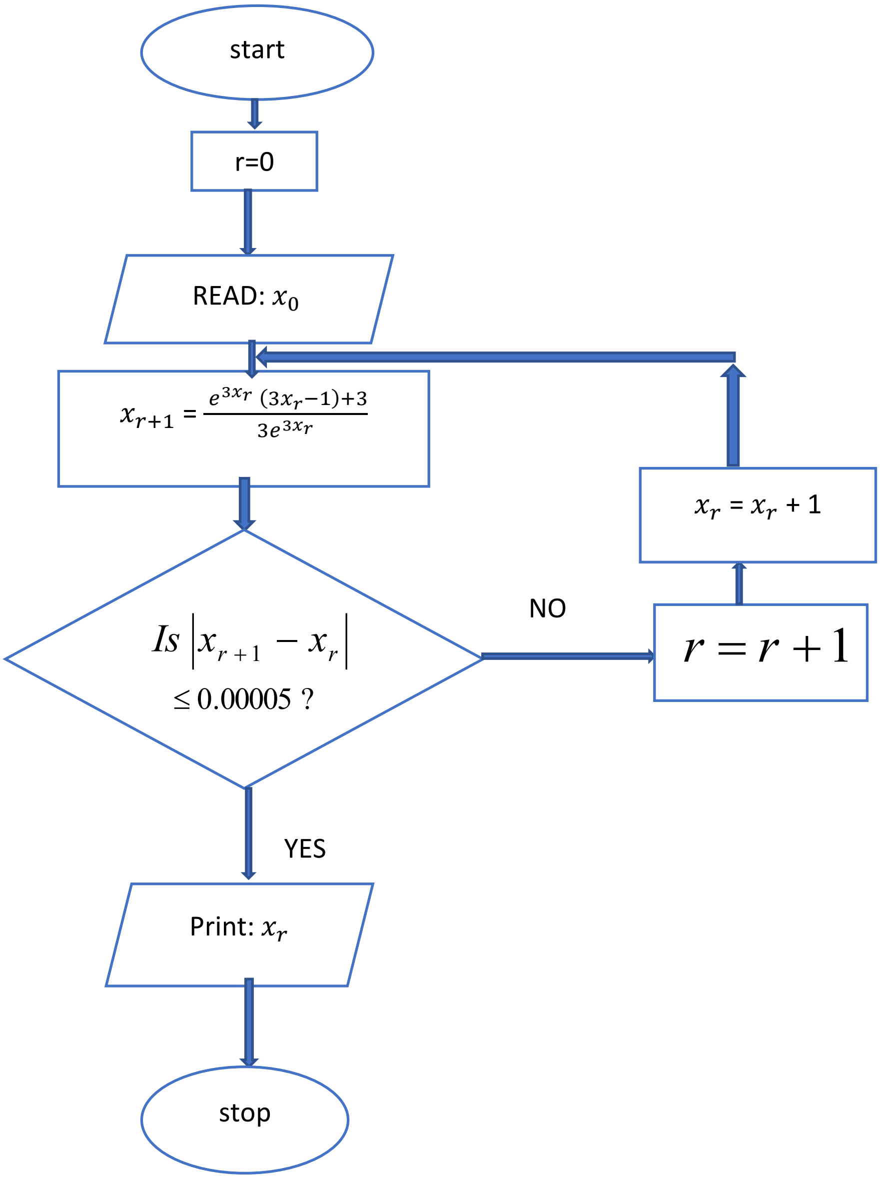
$$\left|\frac{\Delta v}{v}\right| \le \left|2\frac{\Delta r}{r}\right| + \left|\frac{\Delta h}{h}\right|$$

$$\boldsymbol{Maximum\ possible\ error} = \left|\frac{\boldsymbol{\Delta h}}{\boldsymbol{h}}\right| + \mathbf{2}\left|\frac{\boldsymbol{\Delta r}}{\boldsymbol{r}}\right|$$

36. (a) $3x = ln3 \quad e^{3X} = 3$

$let\ f(x) = e^{3X}-3 \quad f^{1}(x)=3e^{3X}$

$$X_{r+1} = X_r - \left(\frac{e^{3X_r-3}}{3e^{3x_r}}\right) = \frac{3x_r e^{3x_r} - e^{3x_r} + 3}{3e^{3x_r}} = \frac{1}{3}\left[\frac{e^{3x_r}(3x_r-1)+3}{e^{x_r}}\right]$$



r	x_0	x_{r+1}	$\lvert x_{r+1} - x_r \rvert$
0	0.33333	0.36788	0.03455
1	0.36788	0.36621	0.00167
2	0.36621	0.36620	0.00001

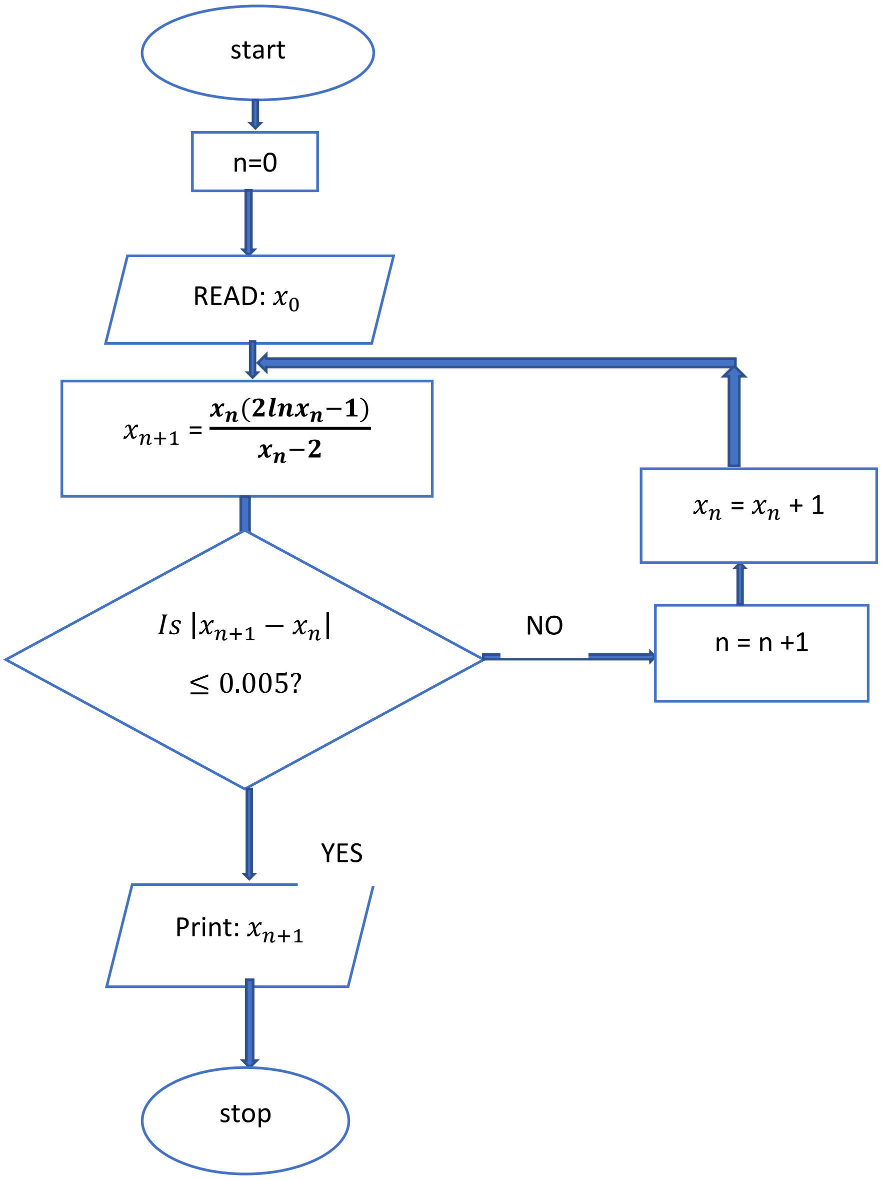
Hence $\boldsymbol{x_{root}}$ = **0.3662**

37. (a) $Let\ f(x) = x - 1 - 2lnx \quad \rightarrow \quad f^1(x) = 1 - \frac{2}{x}$

$x_{n+1} = x_n - \left(\frac{x_n - 1 - 2lnx_n}{1 - \frac{2}{x_n}}\right) = \frac{x_n - 2 - (x_n - 1 - 2lnx_n)}{\frac{x_n - 2}{x_n}} = \frac{x_n^2 - 2x_n - (x_n^2 - x_n - 2x_n lnx_n)}{x_n - 2}$

$$= \frac{x_n^2 - 2x_n - x_n^2 + x_n + 2x_n lnx_n}{x_n - 2}$$

$$= \frac{\boldsymbol{x_n(2lnx_n - 1)}}{\boldsymbol{x_n - 2}}$$



END